BARI-TH/94-180
June 1994

D^* radiative decays and strong coupling of heavy mesons with soft pions in a QCD relativistic potential model

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ABSTRACT

In the framework of a QCD inspired relativistic potential model, we evaluate radiative decay rates of heavy mesons and their coupling with soft pions. The agreement with the existing experimental data is satisfactory. In the limit $m_Q \to \infty$ one obtains results in agreement with the Heavy Quark Effective Theory and is able to predict the values of the relevant couplings; in particular, for the scaled $B^* B\pi$ strong coupling constant g, we find that the non relativistic constituent quark model prediction $g = 1$ is modified, by the inclusion of the relativistic effects due to the light quarks, to the value $g = 1/3$, in agreement with recent QCD sum rules determination.

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In this letter we wish to analyze two problems. The first one is the study of the flavour conserving D^* decays:

$$D^* \to D\gamma \;, \tag{1}$$

$$D^* \to D\pi \;. \tag{2}$$

Even though we do not posses a full set of experimental data yet, results from CLEO II collaboration [1] already yield useful information on (1) and (2) and provide constraints on the theoretical models. The second problem addressed by the present letter is the determination of the strong coupling $g_{D^* D\pi}$, defined by:

$$< D^0(k)\, \pi^+(q) | D^{*+}(p,\epsilon) >= g_{D^* D\pi} \epsilon^\mu q_\mu \tag{3}$$

in the limit $m_c \to \infty$. This coupling has been considered by a number of authors [2, 3, 4, 5]. In fact, it is of interest in chiral effective theories of heavy mesons that describe the strong interactions of heavy $(Q\bar{q})$ mesons with chiral Nambu-Goldstone bosons or light vector mesons, as well as their couplings to weak and electromagnetic currents [6, 7, 8, 9, 10, 11].

In the $m_c \to \infty$ limit $g_{D^* D\pi}$ can be written as follows [12]:

$$g_{D^* D\pi} = \frac{2 m_D}{f_\pi} g \tag{4}$$

and one can show that, in the constituent quark model $g \simeq 1$ [13, 14] [2]. However, recent analyses indicate smaller values; for example from the study of the semileptonic $D \to \pi \ell \nu_\ell$ decay one estimates $g = 0.4$ (this value is obtained using a chiral effective theory and performing the limit $m_c \to \infty$ [2], [4]). Also recent QCD sum rules analyses of the coupling (3) in the heavy quark infinite mass limit point to small values (g in the range $0.2 - 0.4$) [3].

Prompted by these results one may wonder if this departure from the naive quark constituent model might be understood as a consequence of the neglect of the relativistic

[2] The value quoted in ref.[13] ($g = 1$) stems from the knowledge of the spin configurations of heavy mesons in the non relativistic quark model, while the authors in ref. [14] find a slightly different value: $g \simeq 0.8$, obtained in a calculation considering mock mesons (see references therein).

motion of the light antiquark $\bar{q}$ inside the meson D. We can test this simple explanation of a low value for g in a well defined model for the heavy mesons interactions, based on a constituent quark picture of the hadrons; the strong interaction between the quarks is described by a QCD inspired potential and the relativistic effects due to the kinematics are included in the wave equation [15, 16, 17].

Let us briefly describe the main features of this model. The heavy hadrons D_a and D^*_a made up by the quark Q and the antiquark $\bar{q}_a$ are described by the states:

$$
\begin{aligned}
|D_a(p)>= \quad & i \quad \frac{\delta_{\alpha\beta}}{\sqrt{3}}\frac{\delta_{rs}}{\sqrt{2}}\int d\vec{k}\; \psi(\vec{k}+x\vec{p},-\vec{k}+(1-x)\vec{p})\times \\
& \times \quad b^\dagger(\vec{k}+x\vec{p},r,\alpha)\; d^\dagger_a(-\vec{k}+(1-x)\vec{p},s,\beta)|0> , \qquad (5)\\
|D^*_a(p,\epsilon)>= \quad & \quad \frac{\delta_{\alpha\beta}}{\sqrt{3}}\frac{(-\epsilon^\mu\sigma_\mu)_{rs}}{\sqrt{2}}\int d\vec{k}\; \psi(\vec{k}+x\vec{p},-\vec{k}+(1-x)\vec{p})\times \\
& \times \quad b^\dagger(\vec{k}+x\vec{p},r,\alpha)\; d^\dagger_a(-\vec{k}+(1-x)\vec{p},s,\beta)|0> , \qquad (6)
\end{aligned}
$$

where α and β are colour indices, r and s are spin indices, $b^\dagger$ and $d^\dagger_a$ are creation operators of the quark Q and the antiquark $\bar{q}_a$, carrying momenta $\vec{k}+x\vec{p}$ and $-\vec{k}+(1-x)\vec{p}$ respectively. The meson wave function $\psi(\vec{k}+x\vec{p},-\vec{k}+(1-x)\vec{p})$ satisfies the Salpeter wave function [18][1], which includes relativistic effects due to the kinematics explicitly:

$$
\begin{aligned}
\Big\{\sqrt{(\vec{k}+x\vec{p})^2+m_Q^2} \quad & + \quad \sqrt{[-\vec{k}+(1-x)\vec{p}]^2+m_{q_a}^2}-\sqrt{m_D^2+\vec{p}^2}\Big\}\psi(\vec{k}+x\vec{p},-\vec{k}+(1-x)\vec{p}) \\
& + \quad \int d\vec{k}' V(\vec{p},\vec{k},\vec{k}')\psi(\vec{k}',\vec{p}-\vec{k}')=0 \qquad (7)
\end{aligned}
$$

which is valid in a moving frame where the meson D (or D^*), having mass m_D, has momentum $\vec{p}$; the wave function ψ is normalized as follows:

$$
\frac{1}{(2\pi)^3}\int d\vec{k}|\psi|^2 = 2\sqrt{m_D^2+\vec{p}^2}\;, \tag{8}
$$

whereas the instantaneous potential V coincides, in the meson rest frame, with the Richardson potential [19] and in the r-space takes the form:

$$
V(r)=\frac{8\pi}{33-2n_f}\Lambda\left(\Lambda r-\frac{f(\Lambda r)}{\Lambda r}\right)\;, \tag{9}
$$

[1]This equation arises from the bound-state Bethe Salpeter equation by considering the instantaneous time approximation and restricting the Fock space to the $Q\bar{q}$ pairs; for more details see [16].

where Λ is a parameter, n_f is the number of flavours and [2]:

$$f(t) = \frac{4}{\pi}\int_0^\infty dq \frac{sin(qt)}{q}\left[\frac{1}{ln(1+q^2)} - \frac{1}{q^2}\right] . \tag{10}$$

In order to avoid unphysical singularities [20], we assume that $V(r)$, near the origin, is constant:

$$V(r) = V(r_M) \qquad \left(r \le r_M = \frac{\lambda}{3m_D}\frac{4\pi}{3}\right) . \tag{11}$$

The values of the parameters, as obtained by fits to meson masses, are as follows: $m_u = m_d = 38\,MeV$; $m_s = 115\,MeV$, $m_c = 1452\,MeV$, $m_b = 4890\,MeV$, $\Lambda = 397\,MeV$, $\lambda = 0.6$. We note that fits of the heavy meson masses do not constrain light quark masses uniquely; in particular one could obtain similar results for the meson masses by considering , e.g. $m_d = m_u \simeq 100\ MeV$ and including a small and negative constant term V_0 in the potential.

In order to compute the amplitudes for the decays (1) and (2), i.e. $g_{D^*D\pi}$ in (3) and g_V given by (e=electron charge):

$$< D^+(k)|J_\mu^{e.m.}|D^{*+}(p,\epsilon) >= g_V\ e\ \epsilon_{\mu\nu\alpha\beta}\ \epsilon^\nu k^\alpha p^\beta\ , \tag{12}$$

we have to express the currents in terms of quark operators. We write:

$$J_\mu^{e.m.} = Q_{ij}\delta_{\alpha\beta}\int \frac{d\vec{q}\,d\vec{q'}}{(2\pi)^3}\left[\frac{m_i m_j}{E_i(\vec{q})E_j(\vec{q'})}\right]^{1/2} \bar{q}_i(\vec{q},r,\alpha)\gamma_\mu q_j(\vec{q'},s,\beta)\ , \tag{13}$$

where $E_i(\vec{q}) = \sqrt{m_i^2 + \vec{q}^2}$, Q_{ij} is the quark charge matrix and q_i, q_j represent the usual quark field operators.

We also consider the axial vector current A_μ, which is obtained from (13) by substituting $\gamma_\mu \to \gamma_\mu\gamma_5$ and Q_{ij} with the appropriate flavour matrix. The matrix element of the axial current can be written as follows:

[2]This potential grows linearly when $r \to \infty$ and follows QCD predictions for small r.

$$
\begin{aligned}
< D^0(k)|A_\mu|D^{*+}(p,\epsilon) >= -i\{ \quad & \epsilon_\mu \quad (m_{D^*}+m_D)A_1(q^2) - \frac{\epsilon\cdot q}{m_D+m_{D^*}}(p+k)_\mu A_2(q^2) \\
- \quad & \frac{\epsilon\cdot q}{q^2} 2m_{D^*} q_\mu [A_3(q^2)-A_0(q^2)]\} \ ,
\end{aligned} \tag{14}
$$

where $2m_{D^*}A_3 = (m_D+m_{D^*})A_1 + (m_{D^*}-m_D)A_2$ $(q=p-k)$.

Taking the derivative of A_μ, we obtain ($J_5 = i\bar{d}\gamma_5 u$):

$$
(m_u+m_d) < D^0(k)|J_5|D^{*+}(p,\epsilon) >= -i(\epsilon\cdot q)\ 2m_{D^*}A_0(q^2) \ . \tag{15}
$$

For small q^2 the matrix element on the l.h.s. of (15) is dominated by the π^+ pole, therefore we obtain, for q^2 small:

$$
g_{D^*D\pi} = \frac{m_\pi^2-q^2}{m_\pi^2}\frac{2m_{D^*}}{f_\pi}A_0(q^2) \ , \tag{16}
$$

which gives the πDD^* coupling in the chiral limit ($q^2=0$):

$$
g_{D^*D\pi} = \frac{2m_{D^*}}{f_\pi}A_0(0) \ . \tag{17}
$$

We can now compute g_V in (12) and $A_0(0)$ in (17) by our model; we closely follow the approach described in ref. [17] to which we refer the interested reader for further details. We obtain:

$$
g_V = \frac{e_Q}{\Lambda_Q} + \frac{e_q}{\Lambda_q} \ , \tag{18}
$$

where:

$$
\begin{aligned}
\Lambda_Q^{-1} &= \frac{1}{2m_D}\int_0^\infty dk|\tilde{u}(k)|^2\frac{1}{E_Q}\left(1-\frac{k^2}{3E_Q(E_Q+m_Q)}\right) \\
\Lambda_q^{-1} &= \frac{1}{2m_D}\int_0^\infty dk|\tilde{u}(k)|^2\frac{1}{E_q}\left(1-\frac{k^2}{3E_q(E_q+m_q)}\right) \ ,
\end{aligned} \tag{19}
$$

where e_Q, e_q are the heavy and light quark electric charges, $E_j = \sqrt{k^2 + m_j^2}$ ($j = q, Q$ and we put $m_u = m_d = m_q$) and $\tilde{u}(k)$ is related to the wave function ψ of eq. (7) with $\vec{p} = 0$ by the equation:

$$\tilde{u}(k) = \frac{k\ \psi(k)}{\sqrt{2}\pi}\ . \tag{20}$$

$\tilde{u}(k)$ can be obtained by solving (7) numerically by the Multhopp method [21]. In computing eqs. (18, 19) we have put $m_D = m_{D^*} = 2.025\ GeV$ which is the theoretical value obtained by solving eq. (7). As for the other channels, we have put: $m_B = m_{B^*} = 5.33\, GeV$ $m_{D_s} = m_{D_s^*} = 2.05\, GeV$ $m_{B_s} = m_{B_s^*} = 5.366\, GeV$. Λ_Q and Λ_q are the mass parameters whose values are reported in Table I. It may be useful to compare (18) with the result of the constituent quark model [22], where: $\Lambda_q = m_d = m_u = 0.335\ GeV$ or $\Lambda_q = m_s = 0.45\ GeV$ and $\Lambda_Q = m_c = 1.84\ GeV$ for the charm case.

Let us now consider $g_{D^*D\pi}$ as given by eq. (17). Let us observe that in the present approach one obtains the value of the form factors at $q^2 = q_m^2 = (m_{D^*} - m_D)^2$ (or $(m_{B^*} - m_B)^2$), whereas we actually need $A_0(0)$; since for small q^2 $A_0(q^2)$ has a pole at the pion mass, this approximation could in principle introduce relevant differences, expecially for the D case. However we observe that by choosing the potential (9), we have neglected spin-spin terms, which corresponds to the approximation $m_D = m_{D^*}$; for consistency we have therefore to put $q_m^2 = 0$ as well, which is what we shall do. Using the tecniques of ref. [17] we obtain the following equation for the form factor A_1 in (14):

$$A_1(q_m^2) = \frac{1}{4m_D}\int_0^\infty dk|\tilde{u}(k)|^2\left(\frac{E_q + m_q}{E_q}\right)\left[1 - \frac{k^2}{3(E_q + m_q)^2}\right]\ , \tag{21}$$

Moreover, one has:

$$A_3(q^2) = \frac{m_D + m_{D^*}}{2m_{D^*}}A_1(q^2) - \frac{m_D - m_{D^*}}{2m_{D^*}}A_2(q^2)\ ; \tag{22}$$

$$A_3(0) = A_0(0)\ ; \tag{23}$$

therefore, in the limit $m_D = m_{D^*}$ one gets:

$$A_3(0) = A_1(0)\ , \tag{24}$$

provided that $A_2(0)/m_Q^2$ goes to zero when $m_c \to \infty$, which we have tested numerically for $m_Q \gg m_c$ using relations in ref. [17] [3].

A numerical analyses gives the result:

$$A_0(0) = 0.40 \qquad\qquad (D \;\; case) \tag{25}$$

$$A_0(0) = 0.393 \qquad\qquad (B \;\; case) \tag{26}$$

Our results are therefore, in the chiral limit :

$$g_{D^* D\pi} \simeq 12.3 \;, \tag{27}$$

$$g_{B^* B\pi} \simeq 31.7 \;, \tag{28}$$

which shows a deviation of only 2 % from the scaling result $g_{D^* D\pi}/g_{B^* B\pi} = m_D/m_B$.

Using the results of eqs. (18), (27), (28) and Table I, we are able to compute the widths for the possible strong and radiative D^* and B^* decay channels. These results are reported in Table II together with the available experimental information. We observe that these results compare favourably with the experimental data whenever they are available; they are also in agreement with an analysis of these decays performed by us in the framework of HQET for mesons containing one heavy quark [2]. It should be observed that in the our calculation we had no free parameters.

Let us now discuss the determination of the scaled constant g defined by (4):

$$g = A_0(0) = A_1(0) = \frac{1}{4m_D} \int_0^\infty dk |\tilde{u}(k)|^2 \frac{E_q + m_q}{E_q} \left[1 - \frac{k^2}{3(E_q + m_q)^2} \right] \;. \tag{29}$$

It is interesting to consider the non-relativistic limit, where: $E_q \simeq m_q \gg k$. In this limit one obtains:

$$g = \frac{1}{2m_D} \int_0^\infty dk |\tilde{u}(k)|^2 = 1 \tag{30}$$

[3]We have used the analogous of eq.(36) of [17]; one obtains eq. (24) provided that the difference between x_{D^*} and x_D (see (5), (6) for their definition), does not go zero more rapidly than $1/m_Q$.

because of the normalization condition (8). Eq. (30) reproduces the well known constituent quark model result [13, 14].

Let us now take in (29) the limit $m_q \to 0$, which is possible since we work in the chiral limit and there is no restriction to the values of m_q in the Salpeter equation (incidentally our fit of the meson masses is obtained with the rather small value $m_q = 38\ MeV$). In this case, we obtain:

$$g = \frac{1}{3}\ . \tag{31}$$

It is worth to stress that the strong reduction of the value of g from the naive non relativistic quark constituent model value $g = 1$ (eq.(30))to the result (31) has a simple explanation in the effect of the relativistic kinematics taken into account by the Salpeter equation. Furthermore, our asymptotic value of g is in the range of values ($g = 0.2 - 0.4$) determined in [3] by means of QCD sum rules in the limit of infinite heavy quark mass and assuming massless light quarks, that is in the same approximation that we have used in (31).

In conclusion, the potential model has allowed us to describe the radiative decays of heavy mesons, obtaining a rather good agreement with other similar approaches and with existing experimental data. Moreover, it has given us the possibility to give a simple explanation, based on the relativistic kinematics, for the small value of the strong coupling constant g of (3), (4) predicted by QCD sum rules in the $m_Q \to \infty$ and its deviation from the non relativistic quark model result.

Acknowledgments

We thank N.Paver for interesting discussions.

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Table Captions

Table I Mass parameters of eq. (18) (values in GeV).

Table II Radiative and hadronic decay widths of B^* and D^* mesons.

Table I

Decay mode	Λ_Q	Λ_q
$D^* \to D\gamma$	1.57	0.48
$D_s^* \to D_s\gamma$	1.58	0.497
$B^* \to B\gamma$	4.93	0.59
$B_s^* \to B_s\gamma$	4.98	0.66

Table II

Decay rate/ BR	theory	experiment
$\Gamma(D^{*+})$	46.21 KeV	<131 KeV [24]
$BR(D^{*+} \to D^+\pi^0)$	31.3%	$30.8 \pm 0.4 \pm 0.8\%$
$BR(D^{*+} \to D^0\pi^+)$	67.7%	$68.1 \pm 1.0 \pm 1.3\%$
$BR(D^{*+} \to D^+\gamma)$	1.0%	$1.1 \pm 1.4 \pm 1.6\%$
$\Gamma(D^{*0})$	41.6 KeV	
$BR(D^{*0} \to D^0\pi^0)$	50.0%	$63.6 \pm 2.3 \pm 3.3\%$
$BR(D^{*0} \to D^0\gamma)$	50.0%	$36.4 \pm 2.3 \pm 3.3\%$
$\Gamma(D_s^*) = \Gamma(D_s^* \to D_s\gamma)$	0.382 KeV	
$\Gamma(B^{*-}) = \Gamma(B^{*-} \to B^-\gamma)$	0.243 KeV	
$\Gamma(B^{*0}) = \Gamma(B^{*0} \to B^0\gamma)$	9.2 10^{-2} KeV	
$\Gamma(B_s^*) = \Gamma(B_s^* \to B_s\gamma)$	8.0 10^{-2} KeV	